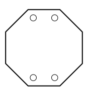
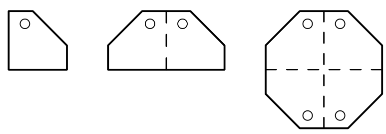
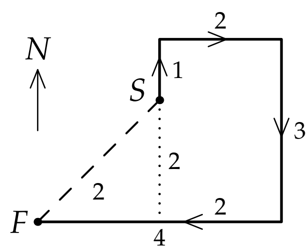
Primary Mathematics Challenge – November 2017

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **D** ($40 \div 2 = 20$) P2 **D** (over twice as high as a human = 5 m)

1	**B**	43 033	This figure was correct in early 2017, but by mid-July it had risen to 43 404 (www.cloudappreciationsociety.org).
2	**C**	trapezium	Each quadrilateral has one pair of parallel sides and is therefore a trapezium — in fact, because the hexagon is regular, the three shorter sides are equal in length, and so each is an *isosceles* trapezium.
3	**A**		The best way to check this is to turn the page over and hold it up to the light!
4	**E**	365	The clock shows 20:17 once each day, and will do so 365 times during 2017.
5	**C**	2019	To check whether a number is a multiple of three, find the total of the digits and if that is a multiple of 3 then the original number is: $2 + 0 + 1 + 9 = 12 = 3 \times 4$.
6	**D**	£15.50	The total in pence is $20 \times 15 + 25 \times 20 + 30 \times 25 = 300 + 500 + 750 = 1550$.
7	**D**	14:02	Seventeen minutes after 13:45 is 14:02.
8	**B**	(5, 4)	If point P has coordinates (x, y), x and y are the mean of 2 and 8, and 2 and 6 respectively. Hence $P = (5, 4)$.
9	**E**	200	Recalling that 1 kg = 1000 g, the number of beans is $500 \div 2.5 = 1000 \div 5 = 200$.
10	**C**	£40	Ms Plummet collects 30 × £12.50 = £375. The difference is therefore £$(415 - 375) =$ £40.
11	**D**	67	When Granddad would have claimed to be 49 he was actually 51, 48 when he was actually 52 — in every case his claimed age and his real age have a total of 100. So this year his age is $100 - 33 = 67$.
12	**B**		Unfolding once and then again will produce these shapes: 
13	**C**	19	The three ages have a total of four times Barry's age. So Barry is $76 \div 4 = 19$.
14	**D**	20	Sleeping for 50 minutes every hour through the day is $\frac{5}{6}$ of every hour and thus $\frac{5}{6}$ of the day. Of 24 hours, $\frac{5}{6}$ is 20 hours.
15	**B**		Pupils should estimate that 23.4 is around two-thirds of 37, and certainly a little more than half of it, so that option B is the only viable one.
16	**A**	North East	We can plot Runaround Sue's path in the diagram on the right. Sue's starting point (S) is 2 km east and 2 km north of her final known position (F). So she must run directly north east.
17	**A**	O J L P M Q K	The order of the squares from the top is N, K, Q, M, P, L, J and O. Hence the first seven to be placed were O, J, L, P, M, Q and K.

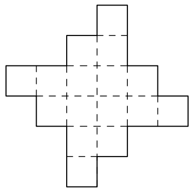


18 **E** $\frac{1}{6}$ After I eat one half, half of the apple pie is left. Eating two thirds of this half leaves one third of one half of the pie, which is one sixth, for Monday.

19 **E** 6, 7, 8, 9 If all of the numbers in one of the sets give a remainder of 1 when divided into 2017, then they are all factors of 2016. We could check each of the numbers 1 to 9 as factors of 2016, but observing that 5 is not a factor of 2016, eliminates all but option E.

20 **C** 2p The sum of £200 is not a multiple of 6, either in pounds or pence; the closest multiple of 6 in pence less than £200 is 199.98p. So the speaker must give away 2 pence.

21 72 cm It is possible to dissect the shape into 16 squares, as shown on the right. Given that the area of the whole 20-sided shape is $144\,\text{cm}^2$, the area of each square is $144 \div 16 = 9\,\text{cm}^2$, and so the side length of each square is 3 cm. Hence the perimeter of the whole shape is $4 \times 2 + 16 = 24$ of these square side lengths: $24 \times 3 = 72$ cm.



22 720 If you start to arrange the numbers you will find that not only do the lists that have 2 and 3 together or 3 and 4 together satisfy the question, but in fact every arrangement does because 6 itself will always form a multiple of 6 no matter which other number it is next to. So the problem is thus to find the number of arrangements of the numbers 1, 2, 3, 4, 5 and 6. We do that in the following way: there are 6 choices for the first number, and once this is chosen there are 5 remaining choices for the second in the list. For each of these 30 choices, there are 4 choices, and subsequently 3, then 2 and then 1 (effectively no choice) for the rest of the numbers in the list. So there are altogether $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ possible lists.

23 535 Labelling the digits of the 3-digit and 4-digit palindromes as "XYX" and "JKKJ" and setting out the addition in columns, we have:

$$\begin{array}{cccc} 2 & 0 & 1 & 7 \\ + & \mathrm{X} & \mathrm{Y} & \mathrm{X} \\ \hline \mathrm{J} & \mathrm{K} & \mathrm{K} & \mathrm{J} \\ \hline \end{array}$$

Since we are adding less than 1000 to 2017, we know that J can stand for either 2 or 3. If J = 2 then X = 5 (as then 7 + 5 has a units digit of 2). If J = 3 then X = 6 (as 7 + 6 has a units digit of 3). Thus:

$$\begin{array}{cccc} 2 & 0 & 1 & 7 \\ + & 5 & \mathrm{Y} & 5 \\ \hline 2 & \mathrm{K} & \mathrm{K} & 2 \\ \hline & & {\scriptstyle 1} & \end{array} \qquad \text{or} \qquad \begin{array}{cccc} 2 & 0 & 1 & 7 \\ + & 6 & \mathrm{Y} & 6 \\ \hline 3 & \mathrm{K} & \mathrm{K} & 3 \\ \hline & & {\scriptstyle 1} & \end{array}$$

In the case of J = 2, from the hundreds digit we can see that K is either 5 or 6 (depending on whether there is a "carry" from the tens column), and so Y is either 3 or 4. However, if K = 6, we would have Y = 4 from the tens column (with the carry of 1 from the units column) and then there would be no carry to the hundreds column. So "XYX" = 535.

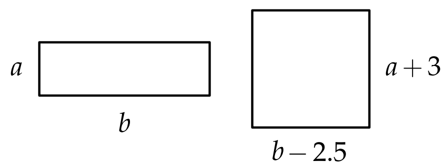
In the case of J = 3, it is evident that the total will never reach over 3000. Hence there is only one answer: 535.

24 36 Since the pie-chart originally showed an angle of 90°, we know that a quarter of the packets were of Plain Crisps (PC). We can now form a table to work out what numbers of packets will also allow 4 packets to be eaten so that the remaining packets make a sixth (60° out of 360°) of all the packets that are left:

Packets to start with		Packets after four are eaten		
PC packets	Total packets	PC packets	Total packets	Fraction that are PC
5	20	1	16	$\frac{1}{16}$
6	24	2	20	$\frac{1}{10}$
7	28	3	24	$\frac{1}{8}$
8	32	4	28	$\frac{1}{7}$
9	36	5	32	$\frac{5}{32}$
10	40	6	36	$\frac{1}{6}$

So there must have been 36 packets left at the end. (This can be the only answer as the proportion of Plain Crisp packets continues to increase as the number of PC packets increases beyond 6.)

25 22 cm Let the rectangle's width and length be a cm and b cm respectively. Then the sides of the square are $(b - 2.5)$ cm and $(a + 3)$ cm. From the area of the rectangle we have $ab = 20$, and from the sides of the square $a + 3 = b - 2.5$.



It should not take very long for pupils to find the solution $a = 2.5$ and $b = 8$, whence the perimeter of the square $= 4 \times 5.5 = 22$ cm.

Some notes and possibilities for further problems

P2 The animal kingdom is rife with surprising data: a peregrine falcon might reach speeds of 200 mph in diving to catch its prey; cheetahs and antelopes on land can run up to 60 mph; a blue whale can reach a length of 30 metres and weigh some 180 tonnes; a hummingbird weighs only 2.5 grams and its wings flap 50 times a second; a pistol shrimp produces a very short, loud click that measures 200 decibels, equal to the noise of a packed football stadium.

2 What other shapes can one get when slicing a regular hexagon with a straight line?

3 Which of the letters of the English alphabet look the same on either side of the window?

Which of these capital letters in the Greek alphabet have line or rotational symmetry?

Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

Can you spot a few different symmetrical letters in the Russian Cyrillic alphabet?

А Б В Г Д Е Ё Ж З И Й К Л М Н О П Р С Т У Ф Х Ц Ч Ш Щ Ъ Ы Ь Э Ю Я

However, the Hebrew alphabet has none:

א ב ג ד ה ו ז ח נ ן ס ע פ ף צ ץ ק ר ש שׁ שׂ ת

Chinese characters are quite often very complex, but many of them have some element of symmetry:

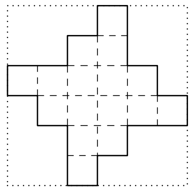
羊 *sheep* 苜 *clover* 桑 *mulberry* 晶 *crystal* 杏 *apricot* 黑 *black* 王 *king* 田 *field* 堂 *hall*
冒 *hat* 山 *mountain* 草 *grass* 父 *father* 童 *child* 谷 *valley* 豆 *bean* 目 *eye* 米 *rice*

How easy is it to read text where the letters are reflected, rotated or have their order reversed – or all three? Here are some well-known mathematical words:

MAЯGOLƎLLAЯAꟼ ƧYMMETЯICAL NOITALUCLAC ECVTNECREP

12 Loosely related to this question is a piece of mathematics that deserves to be more widely known: the *Fold-and-Cut Theorem*, that *any* straight-sided two-dimensional shape can be cut from a single sheet of paper, by folding it (possibly more than once and in several directions) and then making a single straight cut in the folded sheet. If you are ingenious enough, you can even get holes in the resulting shapes. There is an excellent video demonstrating much of this on the Numberphile site: www.numberphile.com/videos/fold_and_cut.html

16 This is similar (a little) to the following conundrum: if Sue runs south 1 km, then east 1 km and then north 1 km she will find that she is back where she started — what might she be able to see: polar bears or penguins?



19 Which two numbers give no remainder when divided into 2017?

21 The diagram on the right shows that the 20-sided shape has the same perimeter as the square that fits exactly around it — can you see why this is?

22 Another problem with the same set of numbers might be to count the lists where a neighbouring pair have a product which is a multiple of 9. This is more difficult to work out as you will get a multiple of 9 only when 3 and 6 are neighbours. We consider first when 3 is followed by 6: there are four other numbers which could go anywhere around the "3, 6" pair, and they can be arranged (using the same method as in the original problem) in $4 \times 3 \times 2 \times 1 = 24$ different ways; we can see there are five positions in which the "3, 6" pair could appear — in this way there are so far $24 \times 5 = 120$ possible lists (with 3 followed by 6). We can use the same reasoning to get 120 (completely different) lists with 6 followed by 3. Therefore there are $120 + 120 = 240$ different lists having a pair whose product is a multiple of 9.

What about those lists with a neighbouring pair whose product is a multiple of 4?

23 Can you find the 3-digit palindrome that one can *subtract* from 2017 to leave a 4-digit palindrome?